Analysis of Dynamic Stress in an Inflating Parachute

K. J. Asfour*
TRW Systems, Redondo Beach, Calif.

Analyses of maximum stresses in parachute canopies have previously associated maximum stress with opening shock. Loads calculated from such analyses are too small to account for many cases of canopy failure. It is proposed that the maximum stress on a canopy is related to the radial velocity parallel to the plane of the skirt with which each concentric ring of the canopy reaches its maximum diameter. At this instant it must decelerate the radial component of the air inflow. This deceleration results in a transient hoop stress imposed on the ring of cloth. An expression is developed for evaluating this stress, designated snap stress, relating it to diameter and filling time. The significance of this relation is discussed and magnitudes are calculated, showing that it can account for otherwise unexplained test failures. Suggestions are made for means of relieving this snap stress, and recommendations are made for further investigation of this source of canopy stress.

Nomenclature

a = constant exponent
A = constant for a given parachute and deployment conditions
C = constant for a given type and design of parachute
d_p = instantaneous skirt diameter of canopy
D_0 = nominal diameter of canopy
D_p = max skirt diameter of canopy
E = energy absorbed by the canopy cloth
h = distance along gore, measured from vent toward skirt; $H = h$ at skirt
KE = kinetic energy of air mass
L = length of canopy cloth in the circumferential direction
m = mass of air
P = max force on the canopy cloth
r = max radius of canopy at any point along the gore; $R = r$ at skirt
t = elapsed time measured from start of inflation
t_f = filling time for a cross section taken normal to the axis of symmetry; $t_F = t_f$ at skirt
v = instantaneous radial velocity of air mass at radius x
V = instantaneous radial velocity of cloth at r
x = instantaneous radius of air mass element
Y = Young's modulus of elasticity
α, β = dr/dh and dt/dh, respectively
θ = angle measured about the axis of symmetry in a plane normal to it
ρ = density of air
τ = t/t_f

Introduction

DESIGN of parachutes is a largely empirical process, relying to a great extent on previously accumulated test data. Under these circumstances, the designer of a new parachute system with design criteria significantly different from those of existing systems is faced with two major problems. The first of these is estimating the aerodynamic performance (drag coefficients, opening loads, etc.) of the new parachute system and the second is determining its structural integrity. There is a real need for reliable mathematical analyses in both of these areas.[1]

This paper is addressed to the determination of structural integrity. The work was promoted by the lack of any analytical or theoretical explanation of repeated failures of large subsonic parachutes or of the limitation on deployment Mach number (approximately 3) for small supersonic deceleration parachutes. Stress analyses published to date do not provide either 1) the distribution of stresses along the gore of the canopy or 2) stress magnitudes high enough to account for observed failures.

It has invariably been assumed in previous analyses that the maximum stress in the canopy is associated with the opening shock of the parachute. The stress calculation then becomes a process of determining how this force is transmitted to the canopy, taking into account the physical and geometric factors involved. Since these stresses peak during canopy inflation, which is a difficult process to study or analyze, static equilibrium conditions were always assumed for the intermediate stages of inflation. Stresses calculated in this way are not truly dynamic stresses, but rather static stresses calculated for "dynamically frozen" conditions.

This paper presents a new approach to parachute stress analysis. It assumes that the major stress is a result of a dynamic force in a plane perpendicular to that of the opening shock, hence normal to the flight path. The calculated magnitude of this stress compares favorably with test results, indicating that the proposed mathematical model of an inflating parachute canopy is an improved representation of the forces involved.

Theoretical Analysis

Total canopy stress at any time can be viewed as consisting of a steady-state part persisting throughout the performance of the parachute together with a transient dynamic stress, which is experienced only at the time of inflation. This paper will evaluate only the transient dynamic stress, which is believed to be several orders of magnitude higher than the steady-state stress.

The process of inflation of a parachute canopy can be considered to approximate the geometry shown in Fig. 1.[2,3] The maximum diameter of the truncated cone portion, where it joins the spherical portion, increases as the inflation process continues. Taking a section through the canopy normal to the axis of symmetry shows the behavior of the cloth ring in that plane, as indicated in the lower set of sketches. There is an important radial component of the air flow inside the canopy which causes the increase in diameter up to the point where the cloth ring, previously slack and unstressed, reaches its maximum extension. In all but the largest parachutes, this entire process takes place in a fraction of a second.

The actual flowfield of the air inside an inflating canopy is so complex that no attempt has been made to represent it

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* Member of the Technical Staff. Associate Member AIAA.

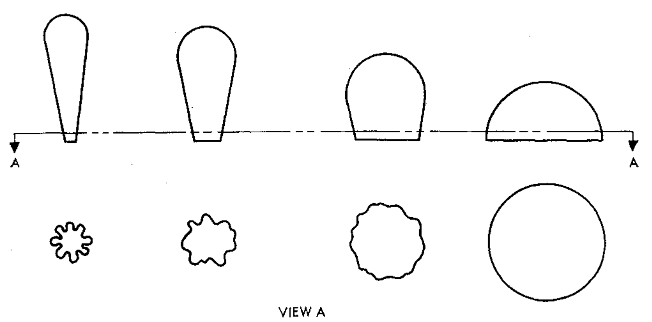

Fig. 1 Geometry of canopy during inflation.

mathematically.[4] Nevertheless, it can be hypothesized that it resembles in some major respects the stagnation condition in three-dimensional flow analyzed by Schlichting.[5] This is the case where "a fluid stream impinges on a wall at right angles to it and flows away radially in all directions." In the case of an inflating parachute, the stagnation plane (i.e., the wall) can be assumed to correspond to the plane between the spherical portion and the truncated cone, or in other words the plane defining the boundary of that portion of the canopy which is already "fully inflated." The air stream impinging on this plane will flow away radially in all directions, carrying with it the canopy material.

Any given cross section of the truncated-cone portion of the canopy, taken normal to the axis of symmetry, will expand as the canopy inflates until it reaches its maximum diameter, at which instant it becomes part of the spherical segment. It experiences at this instant a circumferential dynamic stress due to the fact that it must "stagnate" the radial velocity component of the moving air mass inside the canopy.

It is possible to calculate this circumferential stress by equating the elastic energy-absorbing capacity of the material to the kinetic energy of the air in the radial direction only, since this is the energy that must be absorbed by the canopy at the instant of full inflation. If it is assumed that the elongation vs load characteristic of the canopy material is a straight-line function and if one takes into account the fact that the radial cross section considered is unstressed until it reaches its maximum diameter, then the energy absorbed by the cloth is

$$E = \tfrac{1}{2}P\Delta L \tag{1}$$

If the elastic properties of the cloth obey Hooke's law in accordance with

$$Y = PL/\Delta L \tag{2}$$

and given that $L = 2\pi r$, substitution of Eq. (2) into (1) gives

$$E = \pi r P^2/Y \tag{3}$$

For an estimate of the kinetic energy of the radial component of the moving air mass, the following simplifying assumptions will be made:

1) The canopy cloth will be assumed airtight rather than porous, although this is not actually the case. Air leakage does not exceed 10%[3] and is in fact negligible over the instant of time considered here. This assumption means that the air in the immediate vicinity of the canopy cloth has the same velocity as the cloth itself.

2) The inertia of the cloth itself and any attachments (suspension lines, reinforcements, etc.) will be neglected. In a more detailed analysis this assumption should be evaluated to determine its significance to the end result.

3) Individual gore bulges and unsymmetrical inflation are neglected, thus assuming the parachute to be a perfect body of revolution during the inflation process.

With these assumptions, the kinetic energy of the moving air mass pushing the canopy out in a cross section of unit thickness can be found by writing an expression for the kinetic energy of a differential element at any radius x, then integrating over all values of x and over a radial angle of 2π.

Figure 2 illustrates the procedure. Taking a differential element as shown in the figure, the kinetic energy is given by

$$\Delta \text{KE} = \tfrac{1}{2}v^2 dm = \tfrac{1}{2}v^2(x\cdot d\theta\cdot dx\cdot 1)\rho \tag{4}$$

There remains the determination of v in Eq. (4). In this expression only the velocity in the radial direction is of interest. Assuming stagnation flow as described by Schlichting, the radial component of the air flow will be zero on the axis of symmetry and will increase with increasing distance from the axis (i.e., radius). It can therefore be described by the expression

$$v/V = (x/r)^n \tag{5}$$

Substituting this expression for v in Eq. (4) and summing the total radial kinetic energy gives

$$\text{KE} = \Sigma\Delta\text{KE} = \int_0^r \int_0^{2\pi} \frac{1}{2} V^2 \left(\frac{x}{r}\right)^{2n} x\rho d\theta dx = \frac{\pi}{2n+2}\rho V^2 r^2 \tag{6}$$

Equating the energy absorbed by the cloth [Eq. (3)] to the kinetic energy given by Eq. (6), and solving for P,

$$P = V[\rho Y r/(2n+2)]^{1/2} \tag{7}$$

Discussion

Parameters Affecting Stress

The stress P derived in this way is similar to the transient stresses induced when a whip is cracked or a flag flutters in the wind; in other words, it is a transient stress induced in a slack material when it reaches full extension. In order to distinguish it from other transient stresses in parachute terminology, it will be referred to as the snap stress, since the canopy does in fact snap open, inducing high circumferential stresses in doing so.

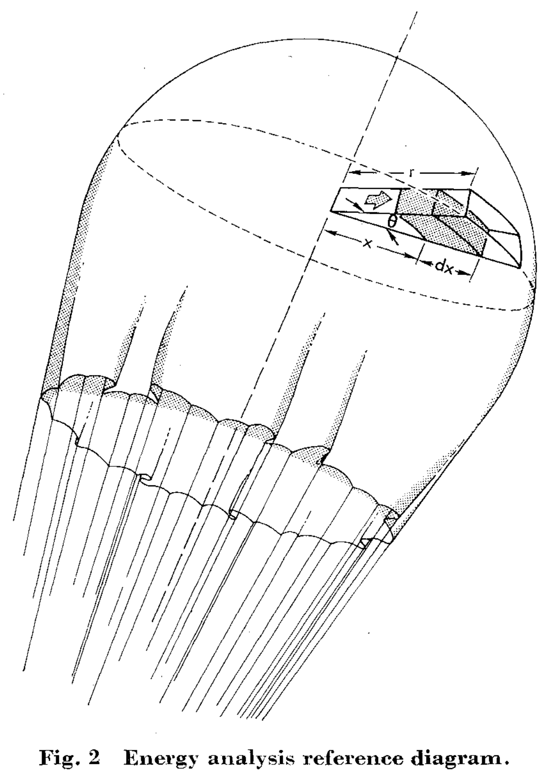

Fig. 2 Energy analysis reference diagram.

The significance of Eq. (7) can be more readily appreciated by evaluating the stress P at the canopy skirt in terms of standard parameters. For a first approximation, the velocity profile mentioned earlier [Eq. (5)] will be assumed linear,[5] i.e., $n = 1$. Considering the inflated canopy, its projected diameter D_p is proportional to the flat canopy diameter D_0; $D_p = CD_0$, where C depends on the type and design of the canopy.†[3] During the inflation process the variation of the instantaneous projected diameter d_p to the fully inflated diameter can be assumed to be of the form

$$d_p/CD_0 = (t/t_f)^a \equiv \tau^a \tag{8}$$

(see Fig. 3).[2] Differentiation of Eq. (8) yields

$$d(d_p)/d\tau = aCD_0\tau^{a-1} \tag{9}$$

At the point of interest ($\tau = 1$, $d_p = D_p$)

$$d(D_p)/d\tau = CD_0a \tag{10}$$

But since $R = D_p/2 = CD_0/2$ and $\tau = t/t_F$, making $d\tau = dt/t_F$, then

$$dR/dt = CD_0a/2t_F \tag{11}$$

Substitution of $CD_0/2$ ($= R$) for r and Eq. (11) for V in Eq. (7) gives, for $n = 1$, the formula

$$P = A\rho^{1/2}D_0^{3/2}/t_F \tag{12}$$

where A is a constant for a particular canopy.

Though the three variables in Eq. (12) obviously interact, it is useful to examine them separately for their effects on canopy stresses. It is clear that the major factors determining snap stress during inflation are filling time (or deployment speed), altitude, and canopy diameter.

Filling time is inversely related to deployment speed, which is therefore directly related to the magnitude of P. It would be expected that higher deployment speeds would lead to greater transient snap stress, and the preceding expression makes it possible to evaluate this effect easily.

Equation (12) indicates that P should decrease with increasing altitude in proportion to $\rho^{1/2}$. In fact, it is generally accepted that t_F decreases with altitude, so that the magnitude of P would be less affected by increasing altitude than would be the case if t_F remained constant.

If a given canopy design maintained a constant t_F regardless of size (a condition which is approached by the Ringsail type), the stress should increase in proportion to $D_0^{3/2}$. For ordinary canopies, however, t_F is linearly proportional to D_0 (Ref. 3), so that a term of the form D_0/t_F could be dropped from Eq. (12),‡ and P would then increase with $D_0^{1/2}$. In any case, it is important to note that large canopies are subjected to greater snap stresses than small canopies under the same loading conditions, and that this stress is not related to the opening shock.

Snap-Stress Distribution

From the design point of view it is important to know where the snap stress maximizes along the gore. Taking any point along the gore as the canopy inflates, the snap stress is given by Eq. (7), which for $n = 1$ can also be written

$$P = \tfrac{1}{2}(\rho Yr)^{1/2}\, dr/dt \tag{13}$$

where dr/dt is the velocity of the cloth at the maximum fully inflated radius of the particular ring under consideration. Referring to Fig. 4, it can be seen that

$$dr/dt = (dr/dh)(dh/dt) = \alpha/\beta \tag{14}$$

For a given parachute under given deployment conditions, the density ρ and the modulus of elasticity Y will be constant. The stresses at two points along the gore h_1 and h_2 can therefore be compared by the expression

$$P_1/P_2 = \alpha_1\beta_2r_1^{1/2}/\alpha_2\beta_1r_2^{1/2} \tag{15}$$

If the curves of Fig. 4 are nondimensionalized with respect to the better-known parameters at the skirt (see Fig. 5), by setting

$$r^* = r/R \qquad h^* = h/H \qquad t^* = t_f/t_F$$

then

$$P_1/P_2 = \alpha_1{}^*\beta_2{}^*r_1{}^{*1/2}/\alpha_2{}^*\beta_1{}^*r_2{}^{*1/2} \tag{16}$$

or more simply, if the conditions with the subscript 2 are taken to be those at the skirt,

$$P^* = \alpha^*\beta^*_{\text{skirt}}\, r^{*1/2}/\alpha^*_{\text{skirt}}\, \beta^* \tag{17}$$

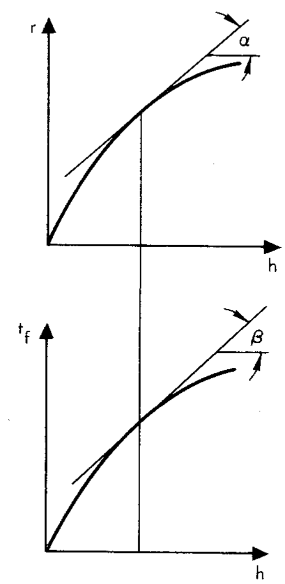

Fig. 4 Maximum radius and time to reach maximum radius along the gore.

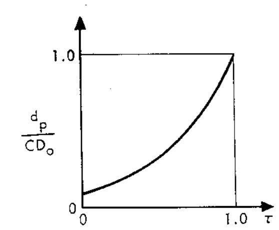

Fig. 3 Variation of skirt diameter during inflation.

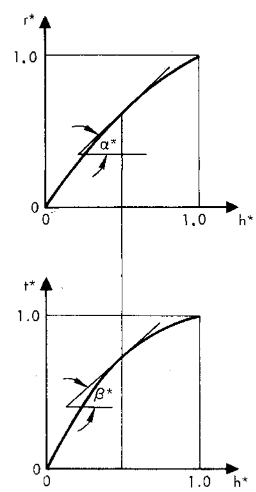

Fig. 5 Nondimensionalized radius and time to reach maximum radius along the gore.

† In this discussion, projected diameter will be taken as equivalent to skirt diameter.

‡ Note that this relates back to deployment speed.

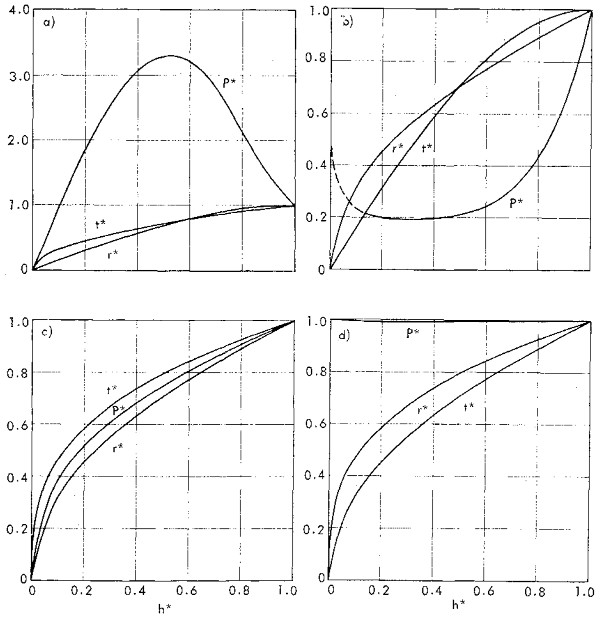

Fig. 6 Snap-stress distribution along the gore for various cases.

In physical terms, this means that the magnitude of the maximum snap stress P depends not only on the radius r, but also on how fast the canopy cloth approaches that radius. The position of maximum P can be determined from the information represented by the curves of Fig. 5, where the variation in r^* represents the steady-state profile of the canopy, and the value of t^* corresponds to the time taken by each concentric ring to reach its maximum radius.

Equation (17) indicates that the point along the gore where the stress ratio P^* reaches a peak is extremely sensitive to the profiles of the r^* and t^* curves. As a demonstration, test data from Ref. 6 was analyzed to determine typical r^* and t^* curves. The general trends determined from this analysis are shown in Fig. 6a, where the corresponding P^* curve is also plotted. The r^* curve represents essentially the profile of the inflated canopy (generally hemispherical for most types). The t^* curve, on the other hand, can vary considerably for different canopies because it is strongly dependent on the total porosity distribution along the gore. The relation of these two curves could assume various forms for different canopies, as shown by the hypothetical cases sketched in Figs. 6b–6d.

The most interesting of these hypothetical cases is the relation shown in Fig. 6d, where the P^* curve has a constant value for all points along the gore. In general, these findings shed a new light on the significance and importance of porosity distribution in parachute design. In fact, they show that the t^* curve could be chosen to make the P^* curve peak at a location preselected by the designer.

Magnitude of Snap Stress

The magnitude of the snap stress given by Eq. (12) can be evaluated by substituting appropriate values from test data in the equation. Table 1 presents a summary of such an evaluation, using the results of limited test data made available to the author and making the assumptions inherent in Eq. (12). In those cases where canopy failure did occur, the snap stress shown in the table was calculated for the ring at which the failure started. In the other cases the stress is calculated at the canopy skirt.

Related Data

An extensive wind-tunnel study made some years ago at the California Institute of Technology[7] included an investigation of the flow about an inflating canopy. Bubbles introduced into the airstream were used to trace the streamlines, and although the report does not include a discussion of flow observed inside the canopy, it states that "The flow field around all the models tested is axially symmetric." Figures 7 and 8, taken from this report, show the streamlines and lines of constant velocity of the flowfield of an inflating canopy. This evidence, together with photographic data on the shape of inflating canopies, supports the assumption that there is an important radial component of airflow within the canopy, parallel to the plane of the skirt.

More recently, a wind-tunnel study performed in Germany for the U.S. Air Force[8] instrumented parachute canopies to determine the pressure distribution. The report of this study refers to the measured air pressure peaks as follows: "The pressure peak occurs first in the canopy vent area and travels very rapidly towards the skirt area. The pressure peaks occur slightly prior to the time at which the canopy reaches its fully inflated shape for the first time . . . they follow very rapidly one another." In one test in this series, the pressure transducers mounted on the canopy were replaced by accelerometers. The results of these measurements are not presented in detail, but the report states that "maximum values were measured on solid cloth flat circular-type canopy models at a location near the canopy skirt, which at the largest deployment speed (160 fps) is accelerated at the beginning of inflation at approximately 50 g's and decelerated at the end of inflation at approximately 200 g's." With the flat diameter of the tested parachute being only 53.5 in., it is clear that the canopy cloth (and the air in its vicinity) acquires a very high radial velocity just prior to reaching the maximum inflated diameter. This deceleration is, of course, a result of the snap force discussed here.

Finally, it has been consistently observed that the diameter of a canopy exceeds its steady-state value for a very brief time after it reaches full inflation, and frequently fluctuates

Table 1 Snap-stress magnitudes

		Conditions at opening				
Canopy type	Nominal diameter, ft	Altitude, 10^3 ft	Dynamic pressure, psf	Material strength, lb/in.	Snap stress, lb/in.	Results of test
Ringsail	189	10	2.5	50	83[a]	Split gore
Ringsail	127	10	20	50	66	Split gore
Ring slot	39	15	135	40	35	Minor damage
Extended skirt	37	20	148	40	30	Minor damage
Solid flat	30	Wind tunnel	69[b]	40	24[a]	No damage
FIST ribbon	9	10	660	500 (Ribbon)	200	No damage but violent pulsation

[a] Stresses calculated by the method of Ref. 2 amounted to less than 1 lb/in.
[b] Velocity rather than dynamic pressure, fps.

through several cycles of expansion and contraction before reaching steady-state condition. In some cases there is a continuous "breathing" of the canopy, continuing indefinitely at a fairly high rate after initial full inflation. It is evident that overinflation in the radial direction parallel to the plane of the skirt must be due to a radial force in that same plane. The overinflation must be accompanied by a proportional stretching of the canopy cloth in the circumferential direction.§ It is suggested that this stretching, frequently on the order of 10%, can be accounted for by the snap stress.

Possibilities for Relieving Snap Stress

If snap stress is a significant cause of canopy failure, it may be possible to find techniques for reducing or absorbing this transient stress, thus eliminating the necessity for making the canopy cloth strong enough to withstand it. Some approaches to alleviating the problem are suggested below.

Reefing

It may be seen that the stress throughout the canopy would be reduced if the radial velocity during inflation were decreased. Reefing is a well-known technique that accomplishes this effect by permitting the canopy to expand in two or more stages rather than one. Continuous disreefing is an obvious solution, and discrete stages would reduce the stress if they were arranged so as to alleviate the stress at the position on the gore where the stress reaches a maximum (or where experience has shown failures to occur). Since failures often occur toward the end of inflation, large reefing ratios would be required in such cases.

Though seldom used at present, permanent reefing might offer another promising solution to the problem of snap stress. The canopy could be permanently reefed, not only at the skirt but all along the gore (i.e., at the different rings) from vent to skirt. Reefing ratios approaching unity would be used in this case. The advantage would be the introduction of a "forced" fullness in the canopy cloth, with the reefing line absorbing the snap stress and relieving the stress in the cloth itself.

Elastic Gore

A standard canopy might be designed with one or more of its gores made of an elastic material calibrated to stretch at somewhat less than the snap-stress loading. In this way the energy could be absorbed before the stress in the cloth reached its nominal maximum.

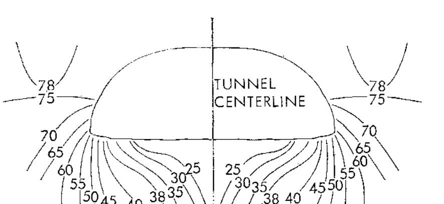

Fig. 7 Distribution of constant velocities (flat circular ribbon parachute, FIST type, 10% geometric porosity, q = 5 psf) from Ref. 7.

§ It may be argued that overinflation beyond the nominal diameter represents a deformation of the hemispherical shape rather than a stretching of the canopy cloth. This would apply only to flat canopies but the same phenomenon is observed with hemispherical designs.

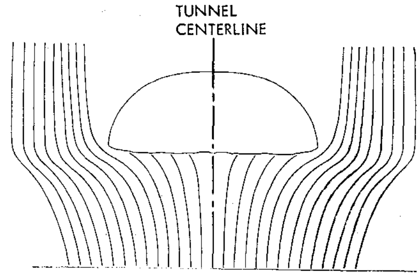

Fig. 8 Streamlines (flat circular ribbon parachute, FIST type, q = 5 psf) from Ref. 7.

Segmented Canopy

The canopy could be divided into two (or more) groups of gores, with the groups held together by elastic bands to form the complete canopy. The elasticity of the bands would allow the canopy to open slightly at maximum inflation, relieving the air pressure inside. The openings would then reclose after peak loading, presenting a normal canopy configuration for steady-state operation. If it were desirable, the gores along the segment lines could be made to overlap to eliminate any loss in drag coefficient during steady-state operation. Aside from relieving the peak pressure loading, the bands themselves would absorb the snap stress. Though not within the present state-of-the-art, this concept is extremely appealing because it allows for design and successful operation of lightweight canopies and at the same time reduces the opening shock of the parachute. An obvious advantage of such a design is the complete elimination of any need for reefing.

Summary and Recommendations

The stress analysis presented here evolved from the need for analysis of canopy failures and explanation of present limitations on parachute use. Impending requirements for high-speed aerodynamic deceleration and low-speed final recovery of large payloads make it essential to understand the stresses to which a parachute is subjected and the parameters affecting those stresses. The empirical methods used to date in parachute design are prohibitively expensive and time-consuming, especially for design of decelerators to operate in the Martian atmosphere.

It has been suggested here that the stresses in a parachute canopy during inflation are much greater than previously supposed. The largest stress has been found to be a dynamic snap stress occurring in each ring of the canopy as it reaches full inflation. A method for evaluating the magnitude and distribution of the snap stress has been proposed.

The exact form of the expression representing the snap stress may differ somewhat from that given here, but evaluation and refinement of this concept should take into account the practical value of increasing accuracy at the expense of simplicity. Further exploration of the present hypothesis might include the following: 1) existing photographic records should be examined to determine in detail the profile and possibly the radial velocity of inflating parachutes; 2) consideration should be given to including the inertia of the parachute material in the analysis; 3) techniques should be devised for direct measurement of the snap stress and radial velocity of a canopy at the instant of maximum inflation; and 4) canopies incorporating a means of relieving snap stress should be tested in order to assess their merit.

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Stability-Augmentation System Gain Determination by Digital Computer

John Frederick Muller*
Martin-Marietta Corporation, Orlando, Fla.

A digital program has been developed to compute the stability-augmentation system gain values yielding the best least squares fit of actual transfer function poles and zeros to desired pole-zero locations, for a set of vehicle transfer functions over a number of flight conditions. The desired locations, the transfer functions to be constrained, the flight conditions to be considered, the control loops to be used, and the gain programming constraints to be observed constitute the program input.

Nomenclature

A	= actual root value
D	= desired root value
KP	= roll-rate feedback to aileron
KR	= yaw-rate feedback to rudder
KCF	= stick crossfeed of roll command into rudder
K_1	= constant
L	= dummy index
M	= number of steps to be used to obtain solution
O	= initial value of actual root
R	= sum of squares
S	= computed step
W	= computed weighing value
X	= dependent variable
$\Delta(\ ), \delta(\ )$	= finite increments in the quantity

Subscripts

i	= flight condition designation
j	= determinant designation (characteristic equation, etc.)
k	= root component designation
l	= dependent variable designation
m	= specific numerical value for the dependent variables

Introduction

A TYPICAL aircraft stability - augmentation problem might be as follows:

1) As nearly as possible, place the complex zero pair of the roll-angle-to-aileron-deflection numerator and the roll subsidence, spiral divergence, and dutch roll roots of the lateral characteristic equation in specific S plane locations.

2) Control these root locations over 28 flight conditions, as follows: Mach number: 0.2, 0.6, 0.8, 0.95, 1.2, 1.4, 1.76; angle of attack: 0°, 18° (at each Mach number); and dynamic pressure: 40, 120 psf (at each Mach number, α combination).

3) Obtain these root locations by scheduling the gains of the following three control loops. KP: roll-rate feedback to aileron; KR: yaw-rate feedback to rudder; and KCF: stick crossfeed of roll command into rudder.

4) Determine the root locations achievable if control gains are scheduled in any of three different ways, i.e.: a) Gains variable with only Mach number; b) gains variable with only angle of attack; and c) gains variable with both Mach number and angle of attack.

Therefore, with the numerator and characteristic equation in LaPlace form, the result is 56 polynomials (numerator and denominator at 28 flight conditions) whose coefficients are functions of numerical constants determined by the flight condition and three dependent variables (KP, KR, and KCF). The problem is to find those values of the dependent variables (for three methods of gain scheduling), which yield the best fit of the actual roots of this set of equations to the desired values. The next section derives the equations to be solved. The remainder of the paper discusses these equations, their mechanization on a digital computer, and the results obtained from applying the program to two example problems.

Derivation of Equations

A typical equation root is symbolized by $(A_{ij1} + jA_{ij2})$ and its desired value by $(D_{ij1} + jD_{ij2})$. The square of the ab-

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* Staff Engineer, Guidance and Control Department.